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MISCONCEPTIONS IN SOLVING SYSTEM OF LINEAR EQUATIONS

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Abstract
Linear algebra is an abstract mathematical field where the concept plays a bigger part compared to the computation itself. The study is to determine common mistakes and misconceptions by undergraduate students in solving System of Linear Equations (SLE) and basis, focuses on elementary row operations. The participants were 30 repeating degree students from the Faculty of Computer and Mathematical Sciences (FSKM), UiTM (Melaka), Jasin Campus who are currently attending the course of Linear Algebra (MAT423). The students were given seven minutes to answer questions on solving the SLE and basis after they have been taught previously in a class. Five mistakes are revealed. Out of five mistakes listed, three of them are identified to be caused by misconceiving the concept of solving SLE and basis. Result of this study suggests that mistakes can be avoided if students are able to use all the concepts correctly.

Keywords: Linear algebra, elementary row operation, system of linear equations, basis, misconceptions.

Introduction

Linear algebra and calculus are major mathematical branches applied in science and technology (Hulya & Nilgun, 2012). For the past two decades linear algebra has become the larger interest in mathematical education and research (Nilgun & Hulya, 2012). Linear algebra is an abstract mathematical field where the concept plays a bigger part compared to the computation itself. Thus, it is not an easy task to enlighten students on this subject.

Most of students are used to direct mathematical calculation (using numbers) as compared to conceptual approach (algebra) even from the earlier education. Therefore, they are unable to relate their previous knowledge to advance mathematic subjects, leading to poor performance in higher education level.

Recently, researchers in mathematical education have been gathering interest in identifying mistakes and misconceptions made by university students. Misconceptions are one of the factors that contribute to student's dissatisfying achievement in academic assessment (Sukru, Betul, & Tevfik, 2011).

Chua (2016) identified the common misconceptions of algebra and proposed the possible solution to overcome the problem. Each problem should be solved with appropriate approaches in order to rectify students' misconceptions on algebra concept. Mulungye (2016) also found certain trend of errors based on misconceptions in algebra. Additionally, the study showed that the teachers' ability and knowledge is required in determining how the mistakes would be built in the whole process of learning in order to improve the method of teaching in class.

Studies on the misconceptions and mistakes committed by students of Politeknik and first intakes university students has been conducted by Siti Aishah and Noor 'Aina (2005) and Hasliza, Noor Azimah and Hafiz Reza (2014) on the basic mathematical concept. They suggested using computers as an aid to improve teaching.

Love, Hodge, Grandgenett, and Swift (2014) compared two methods on teaching linear algebra course; a traditional lecture and a flipped (virtual) style, on the second year students at a mid-sized metropolitan university. They reported the performance of students in the flipped classroom had more improvement compared to the students in the traditional lecture section based on result of the course exams conducted. Teixeira (2015) proposed Peer Instruction (PI) to improve student understanding on the basic concept on linear algebra course, interaction and discussion among peers in classroom. Pre-test was conducted to measure student's understanding before PI was implemented. Based on the pre and post test results, significant increments in students' performance have been observed.

Booth and Koedinger (2008) also used pre and post-test in evaluating conceptual and procedural knowledge of algebra practices by student. The process is done by student answering pre-test and then tutor will provided guided procedural practice of solving linear equation and finally student answering the post-test. The result of this concept is it can improve student learning of correct procedures by immediately feedback from about any error from their tutor.

In a research article by Nilgon and Hulya (2012), six misconceptions have been identified during solving matrix and determinant questions. The focus of the study was on undergraduate students by using qualitative and quantitative methods. They also suggested that the teaching for this subject should be more detailed and all the concepts are clearly defined.

The student faced difficulties when vector spaces were included in the linear algebra courses. Scott (2007), it gives fog observation on student about the vector space. He introduce isomorphic which is as translate, manipulate and back-translate to solve the SLE problem and he introduce r.e.f simplification of the span to solve the span, linearly independent and basis problem. As a result, students are able to solve vector-space problem clearly, easily and fogless.

The understandings on linear algebra courses can be improved by comprehending the concept definition (Berman & Shvartsman, 2016). The same study stated that, understanding the definitions encourages students to learn all concepts of linear algebra in a better way and helps students to change their attitude in learning.

It is agreed that it is essential to be aware on mistakes and misconceptions that might be conceived by students during learning process of linear algebra. The current study aims to determine common mistakes and misconceptions by undergraduate students in solving a linear algebra problem. The participants selected were 30 repeating students from the Faculty of Computer and Mathematical Sciences (FSKM), UiTM (Melaka) Jasin Campus who are currently attending the course of Linear Algebra.

Methodology

The students were given seven minutes to answer 2 questions that use elementary row operations (ERO). The first question is on solving the SLE while the second one involves determining basis of vector space. These students are already being taught on this subtopic previously, though not directly prior to the time when they were given the question.

The questions provided are as follows:

1. Use Gauss-Jordan Elimination to solve the following SLE.

$$\begin{aligned} x_1 \quad\quad\quad &= -2 \\ 2x_1 + 7x_2 &= -11 \\ -x_2 &= 1 \end{aligned}$$

2. Consider the following vectors in P_2. $\bar{r} = 2 + x,\ s = x + 2x^2,\ \bar{t} = 4 + 2x^2$. Show that the set $M = \{\bar{r}, \bar{s}, \bar{t}\}$ is a basis of P_2.

The answers are then collected, marked and analyzed accordingly. Through the answers given, common mistakes are observed and listed.

The suggested solution for question 1 is as follows:

$$\left[\begin{array}{cc|c} 1 & 0 & -2 \\ 2 & 7 & -11 \\ 0 & -1 & 1 \end{array}\right] \xrightarrow{R_2 - 2R_1 \to R_2} \left[\begin{array}{cc|c} 1 & 0 & -2 \\ 0 & 7 & -7 \\ 0 & -1 & 1 \end{array}\right] \xrightarrow{\frac{1}{7}R_2 \to R_2} \left[\begin{array}{cc|c} 1 & 0 & -2 \\ 0 & 1 & -1 \\ 0 & -1 & 1 \end{array}\right] \xrightarrow{R_3 + R_2 \to R_3} \left[\begin{array}{cc|c} 1 & 0 & -2 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{array}\right]$$

$\therefore x_1 = -2;\ x_2 = -1$

The suggested solution for question 2 is as follows:

$$C_1\bar{r} + C_2\bar{s} + C_3\bar{t} = 0$$
$$C_1(2 + x) + C_2\left(x + 2x^2\right) + C_3\left(4 + 2x^2\right) = 0$$

$$\begin{aligned} 2C_1 + 4C_3 &= 0 \\ C_1 + C_2 &= 0 \\ 2C_2 + 2C_3 &= 0 \end{aligned}$$

$$\left[\begin{array}{ccc|c} 2 & \mathbf{0} & 4 & 0 \\ 1 & \mathbf{1} & 0 & 0 \\ 0 & 2 & 2 & 0 \end{array}\right] \xrightarrow{R_1 \leftrightarrow R_2} \left[\begin{array}{ccc|c} 1 & \mathbf{1} & 0 & 0 \\ 2 & \mathbf{0} & 4 & 0 \\ 0 & 2 & 2 & 0 \end{array}\right] \xrightarrow{-2R_1 + R_2 \to R_2} \left[\begin{array}{ccc|c} 1 & \mathbf{1} & 0 & 0 \\ 0 & \mathbf{-2} & 4 & 0 \\ 0 & 2 & 2 & 0 \end{array}\right] \xrightarrow{-\frac{1}{2}R_2 \to R_2} \left[\begin{array}{ccc|c} 1 & 1 & 0 & 0 \\ 0 & 1 & -2 & 0 \\ 0 & 2 & 2 & 0 \end{array}\right]$$

$$\xrightarrow{-2R_2 + R_3 \to R_3} \left[\begin{array}{ccc|c} 1 & 1 & 0 & 0 \\ 0 & 1 & -2 & 0 \\ 0 & 0 & 6 & 0 \end{array}\right] \xrightarrow{\frac{1}{6}R_3 \to R_3} \left[\begin{array}{ccc|c} 1 & 1 & 0 & 0 \\ 0 & 1 & -2 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right]$$

$$\begin{aligned} C_1 + C_2 &= 0 \\ C_2 - 2C_3 &= 0 \\ C_3 &= 0 \\ \therefore C = C_2 = C_3 &= 0 \end{aligned}$$

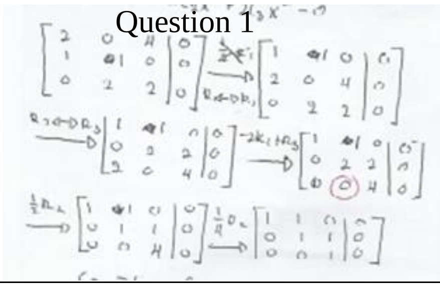
Since $C_1 = C_2 = C_3 = 0$, the system has trivial solution. Thus, the set M is linearly independent. Since the number of vectors in set *M* equal to the dimension of P_2. thus the set M also spans. Therefore, *M* is a basis of P_2.

Findings and Discussion

There are five common mistakes observed from the solutions provided by the students from question 1 and 2 which have been identified as miscalculation (Mistake 1), operational mistakes (Mistake 2), incomplete answer (Mistake 3), wrong method of solution (Mistake 4) and matrix incorrectly transferred from question (Mistake 5).

- Mistake 1: The solution is wrongly calculated. Three of the students solved the question 1 in the correct method but obtained wrong answer because of the computation involving numbers are incorrectly done. For question 2, there are two students doing the same mistake. Such mistake might be caused by punching in the wrong number in calculator or not being careful when doing the calculation.

Question 1



Question 2

Figure 1 Mistake 1

- Mistake 2: Elementary row operation (ERO) incorrectly used. There are three ERO involved to obtained row equivalent matrices. These three ERO can be used in solving the SLE (Harbans, Masita, Rasidah & Zubaidah, 2013).

  - *Operation 1*: interchanging two rows $(R_i \leftrightarrow R_j)$
    Two rows are interchanged, replacing each other's position.

  - *Operation 2*: multiplying row with a non-zero constant $(kR_i \rightarrow R_i)$
    Each element in a row is multiplied with a non-zero constant.

  - *Operation 3:* Additional operation $(kR_i + R_j \rightarrow R_j)$.
    A row that has been multiplied with a non-zero constant is added to another row.

  However, some of the students are confused in using the correct ERO resulting them to resolve in choosing tedious and non-essential steps in solving the SLE. As shown in Figure 2, the student interchanged the second row with the third row. This interchanging process is not necessary because student can just use operation 3 to change the coefficient value of 2 into 0; $((-2)R_1 + R_2 \rightarrow R_2)$.

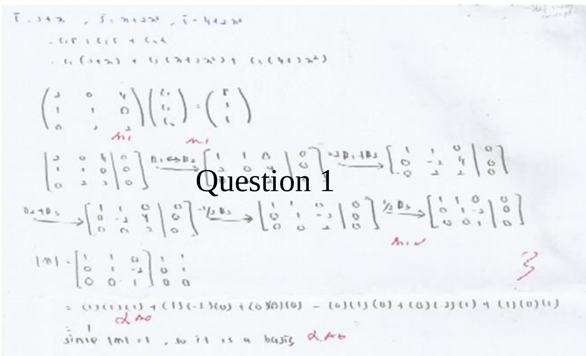
Question 1	Question 2
	

Figure 1 Mistake 2

Another method to solve question 2 is using determinant. Figure 2 show that student were applied two methods of solving; elementary row operations and determinant. However, the methods applied are wrongly used.

- Mistake 3: The answer given is incomplete as shown in Figure 3. Problems on solving SLE require students to obtain the values of unknown variables in the system (which are x_1, x_2 in the question given). Some of the students failed to obtain these values and there are a few causes that lead to the problem unsolved:

  - stopping once the matrix is reduced to reduced row echelon (RRE) form
    Students managed to properly use the right operation to reduce the augmented matrix to RRE form but fail to list out the final solutions, which are the value of x_1 and x_2.

  - stopping halfway before matrix is reduced to RRE
    Students didn’t manage to reduce the augmented matrix to RRE form even though they know how to perform the ERO. A few assumptions can be done on the reason why this happen; the students do not understand why they are performing the operation or they fail to understand that the augmented matrix must be reduced to RRE form.

  - failing to understand the concept of solving equations
    Students are not aware that solving an equation is to find the value of the given variables or unknown, so they fail to give the solution for the given problem.

  - failing to grasp the concept of Gauss-Jordan elimination.
    Students do not understand the concept of Gauss-Jordan elimination causing them to fail on using the method to solve the SLE. This mistake is related to Mistake 4.

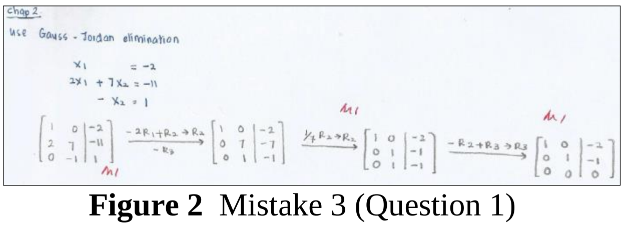


Figure 2 Mistake 3 (Question 1)

- Mistake 4: Wrong method is used to solve the problem. The question clearly stated that the SLE must be solved using Gauss-Jordan Elimination method but some of the students used Gauss Elimination method instead. The matrix is to be reduced to RRE form when solving using Gauss-Jordan Elimination students failed to do so. It is a possibility that they might be confused and not able to differentiate between both methods when solving SLE as illustrated in Figure 4.

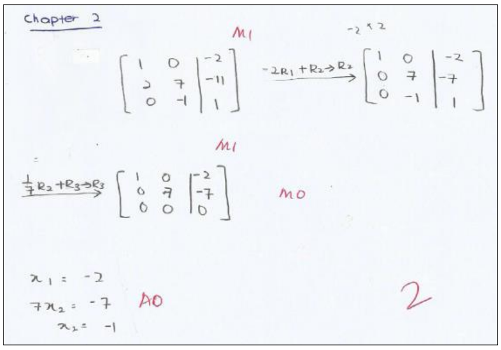


Figure 3 Mistake 4 (Question 1)

- Mistake 5: A set of linear equations is converted into a matrix by taking each equation in the system to be a row of the matrix where the coefficients are kept and the variables are dropped as illustrated in Figure 5. The SLE given in this question would result in an augmented matrix with 3 rows and 2 columns. The wrong solution submitted by a student showed that one value is incorrectly transferred. There are two possibilities that would cause this to happen; a simple careless mistake in transferring the value or not knowing how to represent a system of linear equation in a matrix form.

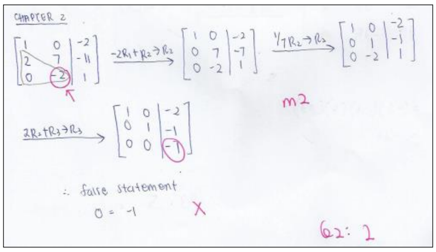


Figure 4 Mistake 5 (Question 1)

For the question 2, the mistakes are usually will lead to the other mistakes made by student. The examples are shows in Figure 6 and 7.

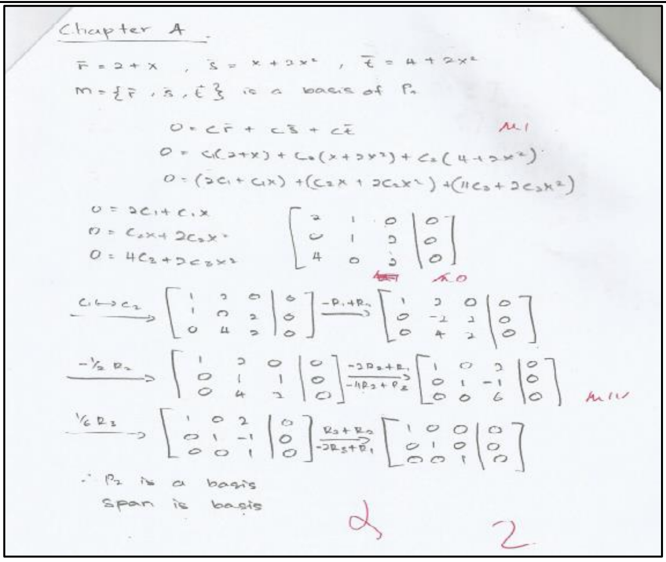


Figure 6 Mistake 3 and Mistake 5 (Question 2)

The student unable to transform the SLE into a matrix form correctly due to the wrong equations obtained as illustrated in Figure 6. The equations are called SLE. The general equation is used by the student to check whether a vector space is linearly independent or not. Since the student checked on the condition of linearly independent only, this will cause on doing Mistake 3. Incomplete conclusion is made.

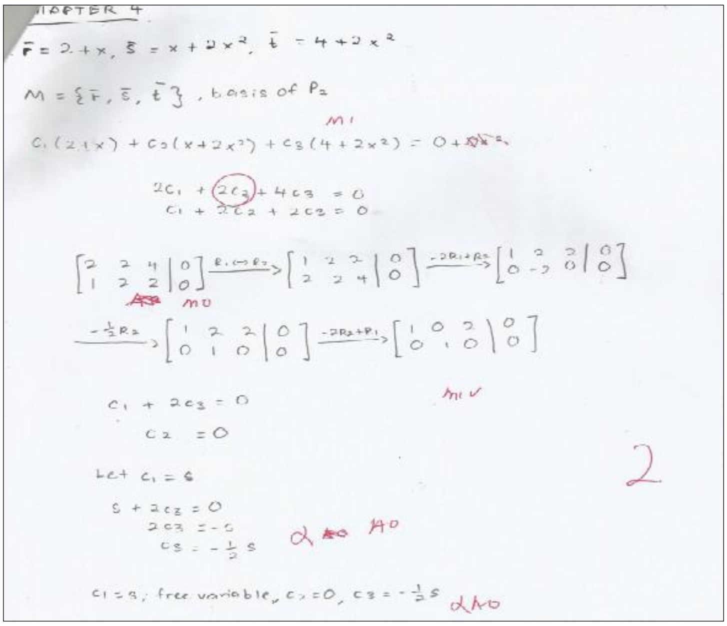


Figure 7 Mistake 4 and Mistake 5 (Question 2)

The matrix obtained is incorrect. The wrong SLE gained because the student extract and get the wrong matrix to solve the problem. This will lead the student to perform the wrong method as shown in Figure 7.

The percentage of students' mistakes on the test can be viewed in Table 1 and Table 2.

Table 1 Percentage of students' mistakes on the question 1

Type of Mistake	No of Students	Percentage (%)
Miscalculation	3	10
Operational mistake	2	6.67
Incomplete answer	7	23.33
Wrong method of solution	4	13.33
Matrix incorrectly transferred	1	3.33

Out of five mistakes listed on Table 1, three of them are identified to be caused by misconceiving the concept of solving the SLE. Those are operational mistake, wrong method of solution, and matrix incorrectly transferred. Two out of these three misconceptions recorded the highest and the second highest frequency as mistakes done by the student. The study shows that misconception takes a large part in affecting the judgment of students in solving SLE. From the findings, it can be implied that Mistake 3 and Mistake 4 are caused by the inability to distinguish between RRE and RE form.

Table 2 Percentage of students' mistakes on the question 2

Type of Mistake	No of Students	Percentage (%)
Miscalculation	2	6.67
Operational mistake	2	6.67
Incomplete answer	7	23.33
Wrong method of solution	5	16.67
Matrix incorrectly transferred	14	46.67

Table 2 show the highest number of mistakes done in question 2 is incorrect transformation of the matrix, which is caused by misconception. The lowest number of mistakes is recorded in miscalculation and operational mistake. From studies, the biggest problem of question 2 is student failed to generate the SLE because they used wrong method of solution. Then they failed to transform the SLE in a matrix form. This problem will lead to mistake 2. From the findings, it can be implied that Mistake 3, Mistake 4 and Mistake 5 are caused by lack of solving concept of basis.

Recommendations

Based on the findings on students' solution, three of the five mistakes were made because of the misconceptions on elementary row operations and basis. It showed that the consideration on misconceptions is important because it leads to mistakes. Result of this study suggests that mistakes can be avoided if students are able to utilize and comprehend all the concepts correctly. Accordingly, lecturers are encouraged to identify potential misconceptions and mistakes in linear algebra and help students to understand the correct procedures in solving SLE. The mistakes can also be reduced if lecturers prepare their lesson plan and find better ways to explain all concepts. For future research, the relationship between method of knowledge delivery in classroom and student performance can be examined. The role of lecturers can be added for this purpose.

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